two-phase flow, in N/m^2; G is the mass flow rate of the gas, in kg/sec; G_S is the mass flow rate of the granular material, in kg/sec; $\mu = G_S/G$ is the mass concentration; β is the volume concentration of the solid phase; ρ is the gas density, in kg/m^3; ρ_S is the true density of the solid-phase material, in kg/m^3; U is the average cross-sectional velocity of the carrier gas, in m/sec; U_S is the average solid-particle velocity, in m/sec; D is the pipe diameter, in m; L is the pipe length, in m; and U_* is the critical velocity, in m/sec. The superscript "0" corresponds to $p_w = 0$ and the subscript "0" corresponds to the pure gas.

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UNSTEADY MASS TRANSFER TO DROPS IN LIQUID FLOW FOR APPRECIABLY
DIFFERENT DIFFUSION COEFFICIENTS OF THE CONTINUOUS
AND DISPERSE GAS PHASES

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Problems of unsteady convective heat and mass exchange between drops and a liquid flow are analyzed under a boundary condition of the "thermal capacitance" type. This condition is substantiated by the method of small perturbations.

Introduction. Problems of unsteady convective heat and mass exchange between drops (particles) and a liquid flow represent the least-studied and most-complicated area in the theory of processes of this class [1, 2]. It has been shown that the phenomenon has several characteristic stages [3, 4], and the necessary transfer equations have been provided [1, 5], which has made it possible to systematize the results achieved in this area for a number of situations of practical importance.

For analysis of the problem of heat- and mass-exchange interaction of drops with the surrounding medium, researchers are forced to resort to simplifications and to make certain

assumptions about the formulation of the principal factors that determine the mathematical model of the process. Here, we shall use at the phase-contact surface a boundary condition of the "thermal capacitance" type [6-8]. Since intuitive premises are often employed in formulations of such a condition, we shall present a possible version of its derivation as applied to the class of problems in question on the basis of methods described elsewhere [8, 9].

1. Boundary Condition of "Thermal Capacitance" Type. We shall examine unsteady convective heat and mass exchange between drops and a continuous medium when the diffusion coefficient of the object material (below, for definiteness, we shall deal only with mass exchange) in the carrier medium D_1 is appreciably smaller than the diffusion coefficient D_2 in a drop ($\kappa = D_1/D_2 \ll 1$). In dimensionless variables, the corresponding boundary-value problem has the form

$$\partial c_1/\partial t + \text{Pe}(\mathbf{v}_1\nabla) c_1 + k_1 w_1(c_1) = \Delta c_1; \tag{1}$$

$$\varkappa [\partial c_2/\partial t + \text{Pe}(\mathbf{v}_2\nabla) c_2 + k_2 w_2(c_2)] = \Delta c_2; \tag{2}$$

$$c_1|_{r \to \infty} \to 0; \quad c_1 = 0, \; c_2 = c_*(\mathbf{r}) \quad \text{for} \quad t = 0; \tag{3}$$

$$c_2 = f(c_1), \quad \varkappa \partial c_1/\partial n = \partial c_2/\partial n \quad \text{on} \quad \gamma, \tag{4}$$

$$c_i = C_i/C_*, \quad k_i = K_i a^2/D_1, \quad i = 1, 2;$$

$$\text{Pe} = ua/D_1, \quad t = D_1\tau/a^2.$$

We shall assume that the velocity fields inside a drop, $\mathbf{v}_2$, and outside, $\mathbf{v}_1$, are known (in general, unsteady) and have scales of the same order of magnitude. The drop shape will be considered unchanged with time. It is clear that for nonzero but constant c_1 values, the problem is easily reduced to (1)-(4) when $r \to \infty$ and $t = 0$.

In derivation of the "thermal capacitance" condition, we shall assume that the dimensionless parameters k_1, k_2, and Pe are independent of κ and have a units order of magnitude. In the equations derived below, further simplifications can be undertaken in certain limiting situations (for example, $\text{Pe} \to \infty$). The first limit expansion (in κ for $\kappa \to 0$) should not be forgotten, i.e., such (repeated) light passages do not always yield valid results.

Assuming that the parameter κ is small, we seek a solution of problem (1)-(4) in the form of an expansion

$$c_j = c_j^0(\mathbf{r}, t) + \varkappa c_j^1(\mathbf{r}, t) + \varkappa^2 c_j^2(\mathbf{r}, t) + \ldots, \quad j = 1, 2, \tag{5}$$

after whose substitution into (1) and (2) we obtain the following equations:

$$\partial c_1^0/\partial t + \text{Pe}(\mathbf{v}_1\nabla) c_1^0 + k_1 w_1(c_1^0) = \Delta c_1^0; \tag{6}$$

$$\Delta c_2^0 = 0; \quad \Delta c_2^1 = \partial c_2^0/\partial t + \text{Pe}(\mathbf{v}_2\nabla) c_2^0 + k_2 w_2(c_2^0), \tag{7}$$

the number of which is sufficient for our purposes. We also have

$$\partial c_2^0/\partial n = 0, \quad c_2^0 = f(c_1^0); \quad \partial c_2^1/\partial n = \partial c_1^0/\partial n \quad \text{on} \quad \gamma. \tag{8}$$

From Eq. (7) for c_2^0 and the corresponding boundary condition (8), it follows that c_2^0 is a function only of time. Then, we integrate the second Eq. (7) over the drop volume and use the Ostrogradskii–Gauss theorem and condition (8). As a result, we find

$$\frac{\partial}{\partial t} f(c_1^0) + k_2 w_2 [f(c_1^0)] = \frac{1}{V} \oint \frac{\partial c_1^0}{\partial n} ds \quad \text{on} \quad \gamma \tag{9}$$

which is the desired condition of the "thermal capacitance" type.

Since (9) contains a derivative with respect to time of a surface function (c_1^0 on γ), information on the initial value of this quantity is required. Note that expansions (5) are external [10, 11], which indicates their suitability at sufficiently large time values. This also corresponds to the intuitive picture of the process [6, 7], when it is assumed that the temperature (concentration) is quickly equalized within the capacitance. Note also that the scales of the variables of the external problem, for which boundary condition (9) serves, are also of practical interest. An internal expansion that has the form of (5) at fixed "internal time" $\bar{t} = t/\kappa$ is used to derive the initial condition for (9). In a zeroth approximation in κ, we have the following problem for the function $\bar{c}_2^0$:

$$\partial \bar{c}_2^0/\partial \bar{t} = \Delta \bar{c}_2^0; \quad \partial \bar{c}_2^0/\partial n|_\gamma = 0, \quad \bar{c}_2^0|_{\bar{t}=0} = c_*(\mathbf{r}), \tag{10}$$

where the overbar indicates quantities that relate to the internal problem. The following relations are sufficient for our purposes (joining with the solution of the external problem):

$$\frac{d}{d\bar{t}} \int_V \bar{c}_2^0 dV = 0 \Rightarrow \int_V \bar{c}_2^0 dV = \int_V c_*(\mathbf{r}) dV, \tag{11}$$

which were obtained after integration of (10) over the entire drop volume and use of the Ostrogradskii—Gauss theorem. Using the principle of limit joining [10, 11] in the second expression of (11) with allowance for the fact that c_2^0 is not a function of the space coordinates and bearing in mind the corresponding condition (8), we find $(\lim_{\bar{t} \to \infty} \bar{c}_2^0 = \lim_{t \to 0} c_2^0)$

$$f(c_1^0) = \frac{1}{V} \int_V c_*(\mathbf{r}) dV \quad \text{at} \quad t = 0 \quad \text{on} \quad \gamma. \tag{12}$$

Thus, the problem of a zeroth approximation in the parameter κ for the function $c_1{}^0$ is completely formulated. This is Eq. (6), which coincides with the starting equation (1), and auxiliary conditions (9) and (12), as well as

$$c_1^0|_{t=0} = 0, \quad c_1^0|_{r \to \infty} \to 0, \tag{13}$$

i.e., we obtain a single-phase rather than a two-phase problem. Below, dealing only with the function $c_1{}^0$, we shall omit its subscript and superscript.

2. Method for Solution of Problems (6), (9), (12), and (13) at $w_1 = c_1$. To solve this problem, we can use the results obtained by analysis of (6) and (13) with the boundary condition $c|_\gamma = 1$, which corresponds to diffusion conditions of surface reaction. A number of studies have been devoted to this problem, whose results we shall use below in the examination of examples. But now we shall assume that the required solution is known. In addition, if the goal is finding only the average Sherwood number or the dependence on time of the amount of object component in a drop, it will be sufficient to have information on the Sherwood number in the problem with a constant boundary value for c.

According to the results of Sec. 1, the concentration on the surface $c|_\gamma$ is a function only of time, which we shall call $G(t)$. Then a solution of problem (6) for a first-order volume reaction, (13) and $c|_\gamma = G(t)$, as is easily obtained by the operational method, has the form

$$c = \frac{\partial}{\partial t} \int_0^t G(\xi) c'(\mathbf{r}, t - \xi) d\xi, \tag{14}$$

where c' corresponds to the problem at G = 1, i.e., a known function. Calculating the derivative with respect to the normal and substituting the result into (9), we find

$$\frac{d}{dt} f[G(t)] + k_2 w_2 \{f[G(t)]\} = -\frac{d}{dt} \int_0^t G(\xi) I(t - \xi) d\xi, \tag{15}$$

where $I = -V^{-1} \int_s (\partial c'/\partial n) ds$ is the integrated flux of material for the auxiliary problem. Equation (15) complements condition (12), where it can be assumed without loss of generality that the right side is equal to unity: f[G(0)] = 1 [or specify G(0) = 1].

In the case of linear functions f and w_2, f(c) = αc and $w_2(c) = c$, which is the solution of Eq. (15) under the initial condition $\alpha G|_{t=0} = 1$, is easily found by the operational method and can be expressed by the Riemann—Mellin integral

$$G(t) = \frac{1}{2\pi i} \int \frac{\exp(pt) dp}{k_2 + p[\alpha + I^*(p)]}, \tag{16}$$

where p is the Laplace-transform variable, and the superscript asterisk indicates transformed values. Integration is performed in (16) along the line $\text{Re } p = \beta_*$, which lies to the right of all singular points of the integrand. The second version, when Eq. (15) can be simply integrated, is the quasi-steady case. In this case, the concentration field outside the drop and, therefore, the flux of material are determined by the steady equation. For

example, this is realized when $Pe \to \infty$ in a boundary-layer approximation with the introduction of "extended" variables $N = n \cdot Pe^{1/2}$ and $T = t \cdot Pe^{1/2}$. In a zeroth approximation in $Pe^{-1/2}$, we have an unsteady relation (9) (in the absence of a source term) and a steady Eq. (6), where the chemical reaction at $k_1 = O(1)$ with respect to $Pe^{-1/2}$ is insignificant in the main approximation as well. A "faster" time variable $T' = t/Pe$ results in an unsteady Eq. (6), but the thermal-capacitance condition in this case gives $\partial c/\partial T' = 0$ on γ, i.e., we arrive at the often examined problem with $c = \text{const}$ on the drop surface. In order to use the thermal-capacitance condition in this version within the scope of our derivation, it is necessary to satisfy the inequality $\kappa Pe \ll 1$, i.e., that the Péclet number be small for D_2 (inside the drop) and large for D_1.

The quasi-steady case is obtained from (14) and (15) as the "fast time" limit, which corresponds to practically instantaneous (in the adopted time scale) transition of the solution of the auxiliary problem to steady conditions. In (14) and (15), in this case, practically over the entire integration interval, $c' = \text{const}(t)$ and $I = \text{const}(t)$, which result, respectively, in the formulas

$$c = G(t) c'(\mathbf{r}),\qquad(17)$$

$$df[G(t)]/dt + k_2 w_2 \{f[G(t)]\} = -G(t) I,\qquad(18)$$

which could also be obtained directly without consideration of time intervals. The solution of Eq. (18) has a simple quadrature

$$t = \int\limits_{G}^{G(0)} f'_G \, dG / \{ IG + k_2 w_2 [f(G)]\}.\qquad(19)$$

Note that in the case in question $I = \text{const} > 0$ and at $k_2 w_2 \geq 0$, the integrand of (19) does not have singular points. In this case, $G < G(0)$ is a monotone decreasing function. A solution of the entire problem is given by formulas (17) and (19) in parametric form in terms of the parameter G.

The problem is simplified in the absence of a chemical reaction inside the drop: $k_2 = 0$. After integration, (15) becomes

$$f[G(t)] - 1 = -\int\limits_0^t G(\xi) I(t - \xi) \, d\xi\qquad(20)$$

which is a nonlinear Volterra integral equation, for whose solution effective numerical algorithms have been developed [12]. It should be noted that the numerical algorithms for determination of the material flux to the drop are fairly complicated for the auxiliary problem. Nevertheless, when they have been constructed, it is relatively simple to add Eq. (15) to them, since it has no effect on $I(t)$.

3. Examples. As an example, we shall examine in a boundary-layer approximation the mass exchange between a spherical drop and the flow past it with a constant velocity at infinity u_∞. If Stokes flow of the liquid is assumed, it is advisable to take $u = u_\infty/(1 + \beta)$ [1], where β is the ratio of the viscosity coefficients of the drop and the liquid surrounding it. The function f will be assumed to be linear: $f = \alpha c$; chemical transition inside or outside the drop will be ignored. Since in the main approximation the principal contribution to the material flux to the drop is made by the region of the diffusion boundary layer (the contribution of the vicinities of the point of infiltration and the edge point, where the equations of the process change form, cannot be taken into account [1]), we shall determine the auxiliary function c' (Sec. 2), which corresponds to the condition $c' = 1$ at $r = 1$ only in that region. Here we can use the method of auxiliary variables [1, 13, 14], which makes it easy to obtain an expression for c' [1, 14]:

$$c' = \text{erfc}\left(\sqrt{\frac{3Pe}{8}} \, \frac{(r-1)(1-\cos\theta)}{\sqrt{2-\cos\theta-\sigma(\theta,\,t)}}\right),\qquad(21)$$

where r, θ are spherical coordinates ($\theta = \pi$ is the point of infiltration);

$$\sigma(\theta,\,t) = 4\frac{3(1-\mu)\exp(t\,Pe) + 1 + \mu}{[(1-\mu)\exp(t\,Pe) + 1 + \mu]^3}, \quad \mu = \cos\theta.\qquad(22)$$

142

Calculation of the total material flux I to the drop gives

$$I = I_s h(t\,\mathrm{Pe}), \quad I_s = 4\sqrt{\frac{2\pi\mathrm{Pe}}{3}}, \quad h(t) = \frac{3}{4}\int_{-1}^{1}\frac{(1-\mu)\,d\mu}{\sqrt{2-\mu-\sigma(\mu,\,t)}} =$$

$$= \left[\mathrm{sh}\left(\frac{t}{2}\right)\right]^{-3/2}\left\{\sqrt{\frac{6z(2z-1)}{2z+1}} + 4z\left[E\left(\sqrt{\frac{1-z}{2}}\right) - \right.\right.$$

$$\left.- E\left(\mathrm{arcctg}\sqrt{z-\frac{1}{2}},\ \sqrt{\frac{1-z}{2}}\right)\right] - \left(2z+\frac{1}{2}\right)\times$$

$$\times F\left[\mathrm{arctg}\sqrt{\frac{2z-1}{z+1}},\ \sqrt{\frac{1-z}{2}}\right]\right\}, \quad z = \frac{1}{2}\mathrm{ch}\left(\frac{t}{2}\right)\ (z<1). \tag{23}$$

At z > 1, the expression for h(t) should be written as

$$h(t) = \left[\mathrm{sh}\left(\frac{t}{2}\right)\right]^{-3/2}\left\{\sqrt{6z(4z^2-1)} + 4zE\left[\mathrm{arctg}\sqrt{z-\frac{1}{2}},\right.\right.$$

$$\left.\sqrt{\frac{z-1}{z+1}}\right] - 4z\sqrt{\frac{3z(2z-1)}{(z+1)(2z+1)}} - \frac{4z+1}{\sqrt{2(1+z)}}\times$$

$$\times F\left[\mathrm{arctg}\sqrt{z-\frac{1}{2}},\ \sqrt{\frac{z-1}{z+1}}\right]\right\}, \tag{24}$$

where $E(z,\,\varphi)$ and $E(z)$ are an elliptical integral of the second kind and a total elliptical integral of the second kind, and $F(z,\varphi)$ is an elliptical integral of the first kind.

Expressions (23) and (24) are fairly cumbersome for use in calculations. At z = 1 ($t_* = 2.63392...$), however, these formulas give

$$h = 3^{-3/4}[2^{1/2} + (3/2)\,\mathrm{arctg}\,(2^{-1/2})] = 1{,}02541..., \tag{25}$$

which makes it possible to check the various approximation formulas used in the literature for approximation of the function h(t). For example, for the expansion in powers of t [1]

$$h(t) = \sqrt{\frac{3}{2t}}\left[1 + \frac{5}{72}t^2 - \frac{7}{1920}t^4 + O(t^6)\right] \tag{26}$$

the error at point t_* is 3.86%, For large times, we can obtain for h(t) the asymptotic formula

$$h(t) = 1 + 3\{3t/2 - 5 + \sqrt{3} + 3\ln(\sqrt{3} - 1)\}\exp(-2t) =$$
$$= 1 + (4{,}5t - 12{,}611)\exp(-2t) + ..., \quad t\to\infty. \tag{27}$$

At point t_*, the error of (27) is 2.86%, but the value of h is less than unity. Formula (27) is qualitatively suitable at t > t_m = 3.30244, when (27) becomes a monotone decreasing function.

Since the example in question corresponds to a boundary-layer approximation (high Pe numbers), according to (22), the use of formula (27) will be consistent. If we limit ourselves to the first term in (27) (units), we obtain a quasi-steady solution (19). The second (unsteady) term in (27) characterizes the transition to a steady solution in the auxiliary problem. Allowance for this term provides a correction for (19), for whose construction it is advisable to use the inequality Pe ≫ 1.

After Laplace transforms of (27) and (20) (f = αG), we arrive at the following expression for G*:

$$G^*(p) = \left\{\alpha p + \lambda\sqrt{\mathrm{Pe}}\left[1 - \frac{12{,}611p}{p+2\,\mathrm{Pe}} + \frac{9p\,\mathrm{Pe}}{2(p+2\,\mathrm{Pe})^2}\right]\right\}^{-1}, \tag{28}$$

$$\lambda = 4\sqrt{\frac{2\pi}{3}}.$$

The function G^* has three poles in the plane p; at $Pe \gg 1$, the values of the first two are on the order of $Pe(p_{1,2} \approx -2Pe)$, while the third is determined by the expression

$$p_3 = -(\lambda/\alpha)Pe^{1/2} - 11{,}486\,(\lambda/\alpha)^2 + O\,(Pe^{-1/2}), \quad Pe \to \infty, \tag{29}$$

i.e., with conversion of (28), the exponential function corresponding to (29) will markedly dominate at large time values. We have

$$G(t) = \frac{1}{\alpha}\left[1 + O\left(\frac{1}{Pe}\right)\right] \times$$
$$\times \exp\left\{-t\left[\frac{\lambda}{\alpha}\,Pe^{1/2} + 11{,}486\,\frac{\lambda^2}{\alpha^2} + O\,(Pe^{-1/2})\right]\right\}. \tag{30}$$

Formula (30) defines more accurately formula (19) with the simplifications corresponding to the example of allowance in a first approximation of the terms that characterize the transition of the mass-exchange process in the auxiliary problem to steady conditions.

The analysis that was performed pertains to solution of the problem in a boundary-layer approximation. At moderate values of the Péclet number, the analytic solution of the auxiliary problem is unknown. However, there exist fairly accurate and simple approximation formulas [15] that can be used in our problem. The relation [15]

$$I = I_s\sqrt{\operatorname{cth}(\xi t)}, \quad \xi = 2u_\infty a/[3D_1(1+\beta)], \tag{31}$$

gives an error for the flux that does not exceed 1% and is approximately the same (with corrections I_s and ξ) for calculation of the material flux to a solid particle. A Laplace transform of (31) results in

$$\frac{I^*}{I_s} = \frac{\sqrt{\pi}}{4\xi}\left\{\frac{\Gamma[p/(4\xi)]}{\Gamma[p/(4\xi) + 1/2]} + \frac{\Gamma[p/(4\xi) + 1/2]}{\Gamma[p/(4\xi) + 1]}\right\}. \tag{32}$$

By the way, this expression can be used to describe the flow of material to a drop in the presence of a volume chemical reaction of the first order [15, p. 56]. A somewhat less accurate but simpler formula [15] is

$$I = I_s[1 - \exp(-\xi t)]^{-1/2}, \tag{33}$$

the Laplace transform of which has the form

$$I^* = I_s\sqrt{\pi}\,\xi^{-1}\Gamma\,(p/\xi)\,\Gamma^{-1}\,(p/\xi + 1/2). \tag{34}$$

Calculation of integral (16) ($k_2 = 0$) by means of contour integration and the use of residue theory lead to the formula

$$G(t) = \frac{1}{y}\sum_{j=1}^{\infty}\exp\,(-\nu_j\xi t)\left\{\nu_j\left[\psi\left(\frac{1}{2} - \nu_j\right) - \psi\,(-\nu_j)\right]\right\}^{-1}, \tag{35}$$

where $\psi(z)$ is the logarithmic derivative of the gamma function [16], and ν_j are roots of the transcendental equation

$$y = \alpha\xi I_s^{-1}\pi^{-1/2} = -\Gamma\,(-\nu)\,\Gamma^{-1}\,(1/2 - \nu). \tag{36}$$

Curves of the first three roots as functions of the parameter y are shown in Fig. 1. Beyond the scope of the graphs, we can use the expansion

$$\nu_{j+1} = j + m_j/y - m_j^2\left\{\ln 4 - \sum_{i=1}^{j}[i\,(2i-1)]^{-1}\right\}/y^2 + \cdots, \quad y \to \infty, \tag{37}$$

where $m_j = 1\cdot 3\dots(2j-1)/(\sqrt{\pi}2^j j!)$, $j \geq 1$; $m_0 = \pi^{-1/2}$; and the sum in (37) at $j = 0$ is equal to zero. Formula (35) is convenient for calculations at large time values. At small time values, we can use the formula

$$G(t) = \alpha^{-1}[1 - 2I_s\alpha^{-1}(t/\xi)^{1/2}], \quad t \to 0, \tag{38}$$

which was derived by means of the asymptotic function [16] for the function $\Gamma(z)$ at $z \to \infty$ in (16).
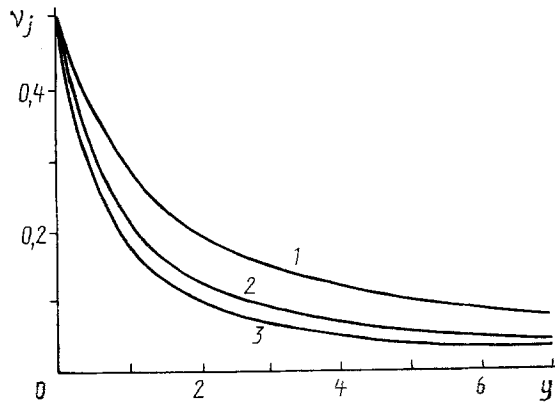
Fig. 1. First roots of Eq. (36): 1) $j = 1$;
2) 2; 3) 3.

Formulas (35) and (38) allow the kinetics of the mass content of a drop to be traced over a wide range of times. Tables [17] can be used to calculate the function $\psi(z)$. Note, too, that a solution for the example represented by formulas (31) and (32) is constructed similarly.

4. Comments. The presence in the problem of several dimensionless parameters whose values can vary within wide limits gives rise to the problems of special study of various limiting versions at $\kappa \ll 1$ and k_1, k_2, and $Pe \gg 1$, which we shall not treat in detail here. The region of expansion nonuniformity can usually be judged after two approxima- tions* (in the given case, with respect to the parameter κ). Nevertheless, certain state- ments can be made concerning the possibility of improvement of the above estimate $\kappa Pe \ll 1$ of the validity of use of a boundary-layer approximation when a "thermal capacitance" boun- dary condition is used; the relation $\kappa Pe \ll 1$ can be weakened somewhat in view of the fact that the term that is proportional to Pe in (2) does not in the main approximation contri- bute to the formation of the thermal-capacitance condition. In fact, if along with Δc_2^0 we take this term into account in the first formula of (7), we obtain the equation $\Delta c_2^0 = Pe(v_2 \nabla c_2^0)$, which (with allowance for the boundary condition of the absence of material flux), just as earlier, is satisfied by a function only of time. In the next formula of (7), such a term is also insignificant after volume integration [see derivation of formula (9)], since for an incompressible liquid it is reduced to a surface integral $\oint v_{2n} \cdot c_2^0 ds$, which is equal to zero, since for a drop of unchanged shape we can assume $v_{2n} = 0$ on γ. Thus, the naturally assumed increase in the rote of the term $Pe(v_2 \nabla) c_2$ with a rise in the Pe number is insignificant for construction of a "thermal capacitance" boundary condition in a first approximation.

The calculation version examined in Sec. 2 refers to an auxiliary problem that is linear outside the drop, which made it possible to write the principal equation for the function $G(t)$ in the form of (15). In the case of a nonlinear function $w_1(c_1)$, the integral in (9) can also be determined by solution of the auxiliary problem, but in this case it should be as- sumed that $c_1|_\gamma = G(t)$, where $G(t)$ is an unknown function. The surface integral in (9) be- comes a nonlinear functional of $G(t)$, whose form can hardly be established by exact methods. In this case, it seems advisable to use new approximation approaches [15, 18] (for example, the method of "carry through" of the Laplace transform) to find an analytic representation of the integral (of material flux) in (9).

NOTATION

a is the characteristic dimension of the drop; C_1, C_2, C_*, and $c_*(r)$ are the concen- trations of the object component in the flow and in the drop, the concentration scale, and the initial value in the drop; $c_j^i(r, t)$ are components of the expansions of the concentra- tions into series in powers of κ; $erfc(z)$ is an auxiliary probability integral; $f(c)$ is a function that establishes a relationship between the concentrations of the continuous and disperse phases at the drop boundary with phase equilibrium; K_i and w_1 (i = 1, 2) are

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*On this topic, see, for example Dil'man and Polyanin [15, p. 24], who discuss the nonuni- formity of an expansion in $Pe^{-1/2}$ for a drop at $\beta \to \infty$.

a proportionality factor (dimension scale) and a dimensionless function that characterizes the chemical transition; n is the normal coordinate; p_i (i = 1, 2, 3) are singular points (poles) of the function $G^*(p)$ (28); r is a brief notation for the set of space coordinates; s is the drop surface; u and u_∞ are the velocity scale and the liquid velocity at a great distance from the drop; α is the distribution coefficient; $\Gamma(z)$ is Euler's gamma function; γ is the coordinate (equation) of the drop surface; τ is time; and V is the dimensionless drop volume.

LITERATURE CITED

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